Magnetohydrodynamic Nozzle

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Let us consider the one-dimensional steady-state motion of a completely ionized gas in a channel of constant cross section, and acted upon by external crossed electric and magnetic fields of constant magnitude. Let

$$\mathbf{u} = (u, 0, 0) \qquad \mathbf{E} = (0, E, 0) \qquad \mathbf{H} = (0, 0, H)$$

Then the system of equations describing the motion takes the form[1] (using conventional symbols)

$$\rho u \frac{du}{dx} + \frac{dp}{dx} = \frac{\sigma H^2}{c^2}\left(c\frac{E}{H} - u\right) \quad \text{(Euler equation)} \quad (1)$$

$$\rho u C_p \frac{dT}{dx} - u\frac{dp}{dx} = \frac{\sigma H^2}{c^2}\left(c\frac{E}{H} - u\right)^2 \quad \text{(energy equation)} \quad (2)$$

$$\rho u = \rho_0 u_0 = G/S \quad \text{(rate equation)} \qquad p = \rho R T \quad \text{(equation of state)} \quad (3)$$

$$u = u_0 \quad \rho = \rho_0 \quad p = p_0 \quad T = T_0 \quad M = M_0 \quad \text{at} \quad x = 0 \quad (4)$$

The conductivity σ is assumed to be finite and scalar; c_p and R are constant.

The conditions for acceleration with a simultaneous increase in the Mach number are[1]:

$$u < u_2 < u_1 < u_3 \quad \text{at} \quad M < 1 \qquad u_1 < u_2 < u < u_3 \quad \text{at} \quad M > 1 \quad (5)$$

Here

$$u_3 = c,\ E/H \qquad u_1 = [(k-1)/k]u_3$$
$$u_2 = \frac{1 + kM^2}{2 + (k-1)M^2}u_1 \qquad k = \frac{C_p}{C_v} \quad (6)$$

It has also been established that a proper selection of the electromagnetic fields can result in continuous acceleration with passage through the speed of sound. In order to achieve a continuous transition through the hydrodynamic speed of sound in a straight channel the following must hold:

$$u = u_1 \quad \text{at} \quad M = 1 \quad (7)$$

We can solve the system (1–4) in the form[4]

$$\frac{2k}{k+1}\frac{\sigma H^2}{c^2\rho_0 u_0}x = \ln\frac{u_3 - u_0}{u_3 - u} + L\left(\frac{k}{u_3}\right)^2\left[\ln\frac{(u_1 - u)(u_3 - u_0)}{(u_3 - u)(u_1 - u_0)} + \frac{u_3}{k}\left(\frac{1}{u_1 - u} - \frac{1}{u_1 - u_0}\right)\right]$$
$$T = \frac{u[(u_3 - u)^2 - 2K]}{2c_p(u_1 - u)} \quad (8)$$

The density and pressure in the stream are determined from Eqs. (3). Let us denote

$$K = \frac{(u_3 - u_0)^2}{2} + c_pT_0 - \frac{u_3}{\rho_0 u_0}p_0$$
$$L = \frac{k-1}{k+1}\left[\left(\frac{u_3}{k}\right)^2 - 2K\right] \quad (9)$$

By means of the expression for T in Eq. (8), the Mach number can be expressed as

$$M^2 = \frac{u^2}{kRT} = \frac{2u(u - u_1)}{-(k-1)(u - u_1)^2 + 2u_1(u - u_1) + Cu_1^2} \quad (10)$$

where C is determined from the boundary condition. It is evident that condition (7) can be satisfied when $C = 0$. The quantity

$$C = \frac{u_0 - u_1}{u_1^2}\left[\frac{2u_0}{M_0^2} + (k-1)(u_0 - u_1) - 2u_1\right]$$

may be equal to zero either at $u_1 = u_0$, or at

$$u_1 = u_1^* = u_0\frac{2 + (k-1)M_0^2}{(k+1)M_0^2} \quad (11)$$

The value of $u_1 = u_0$ is of no interest here, since in this case the inequalities (5) will no longer hold. For

$$u_1 = u_1^* \qquad K = \frac{1}{2}\left(\frac{u_3}{k}\right)^2 \qquad L = 0$$

and Eqs. (8) and (10) can now be simplified to read as follows:

$$x = \frac{k+1}{2k}\frac{c^2}{\sigma H^2}\frac{G}{S}\ln\frac{u_3 - u_0}{u_3 - u} \quad (12)$$

$$T = \frac{u}{2C_p}\left(\frac{k+1}{k}u_3 - u\right)$$
$$M^2 = \frac{2u}{-(k-1)u + (k+1)u_1} \quad (13)$$

According to the latter equation, $U_2 = \frac{1}{2}(u + u_1)$. This means that

$$u < u_2 \quad \text{at} \quad M < 1(u_2 < u_1)$$
$$u > u_2 \quad \text{at} \quad M > 1(u_2 > u_1) \quad (14)$$

Thus, a proper selection of the fields ($u_1 = u_1^*$) not only insures a smooth transition through the speed of sound, but also results in the acceleration of both the subsonic and supersonic streams with a simultaneous increase in the Mach number. The stream velocity, according to (12), is given by

$$u = u_3 - (u_3 - u_0)\exp\left(-\frac{2k}{k+1}\frac{\sigma H^2 S}{c^2 G} - x\right) \quad (15)$$

The critical cross section $x = l$ is determined by substituting $u = u_1$ into (12):

$$l = \frac{k+1}{2k}\frac{Gc^2}{S\sigma H^2}\ln\frac{u_3 - u_0}{u_3 - u_1} \quad (16)$$

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The greater the magnetic field intensity, the shorter the subsonic segment. By assigning a value to l, we can determine the value of H which will assure the transition through the speed of sound, and we can also find the intensity of the corresponding electrical field E from the following equations:

$$H^2 = \frac{k+1}{2k}\frac{Gc^2}{S\sigma l}\ln\frac{u - u_0}{u_3 - u_1}$$

at

$$x = l \qquad E = \frac{k}{k-1}u_1\frac{H}{c} \tag{17}$$

Substituting (17) into (15) we get

$$u = u_3 - (u_3 - u_0)\left(\frac{u_3 - u_0}{u_3 - u_1}\right)^{-\frac{x}{l}} \tag{18}$$

The values $x < l$ correspond to the subsonic portion of the nozzle, whereas the values $x > l$ correspond to the supersonic portion. The maximum velocity u_3 is attained at infinity. At a given terminal velocity u_-, the overall length of the nozzle is

$$L = l\,\frac{\ln[(u_3 - u_0)/u_3 - u_1)]}{\ln[(u_3 - u_0)/u_3 - u_1]} \tag{19}$$

The temperature in the critical cross section is independent of the location of the section

$$T^* = u_1{}^2/kR \tag{20}$$

The maximum Mach number (at infinity), as well as the maximum temperature, are found from Eqs. (13) at $u = u_3$:

$$M_\infty = \sqrt{\frac{2k}{k-1}}$$

$$T_\infty = \frac{u_1{}^2}{2(k-1)R} = \frac{k}{2(k-1)}T^* \tag{21}$$

When accelerating the supersonic stream ($M_0 > 1$), in particular at $u_1 = u_1^*$, the intensity of the magnetic field is determined by Eq. (12) from the known value of the thermal velocity u_- and length of the accelerating section $x = L$:

$$H^2 = \frac{k+1}{2k}\frac{G}{S}\frac{c^2}{\sigma L}\ln\frac{u_3 - u_0}{u_3 - u_-} \tag{22}$$

The velocity along the channel in this case is given by

$$u = u_3 - (u_3 - u_0)\left(\frac{u_3 - u_0}{u_3 - u_-}\right)^{-x/L} \tag{23}$$

Integrating Ohm's law with respect to the length and using the velocity equation (15), we can obtain expressions for the current flowing through the gas and for the electric power consumed in acceleration:

$$I = \frac{k+1}{2k}\frac{G}{b}\frac{c}{H}(u - u_0)$$

$$N = \frac{k+1}{2k}u_3G(u - u_0) \tag{24}$$

where b is the electrode gap.

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Influence of the Earth's Orbital Motion on Radar Measurements of Range and Velocity in Space

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General Propositions

When we measure the range (the distance from the earth) and velocity relative to the earth of a target in space at extreme distances (of the order of one or more astronomical units), we encounter a number of specific effects which become negligible in other cases. The propagation time of radio waves in these conditions reaches values of hundreds or thousands of seconds; in this time the measuring station moves tens of thousands of kilometers with the earth in its orbit. The earth's orbital velocity is about 10^{-4} the velocity of light, so that the effects of the special theory of relativity become noticeable. The presence of the interplanetary (and interstellar) medium influences the velocity of propagation of radio waves: given a path of the order of 1 a.u. or more this can create an appreciable error in determining distance, since we do not know just how much the velocity of propagation of radio waves differs from the velocity of light in a vacuum. In the present paper the writer considers the influence of the earth's orbital motion on the results of radar measurements of velocity and range.

We shall take as our initial values τ, the time delay of a radio signal in its path from the transmitting station to the target to the receiving station, and dT/dt, its time derivative, choosing these values because they are the ones to which we can reduce any actual parameters received from radar stations (delay time, phase delay, Doppler frequency, etc.).

To analyze the influence of the earth's orbital motion we shall find it convenient to consider the process of measurement in an inertial system of coordinates that is stationary in

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